and from Eq. (7)

$$y/x = (\sin\alpha - 1)/\cos\alpha \tag{9}$$

so that the trajectory is a straight line segment. For Fig. 3b the orbits have equal periods, so the resulting trajectory is periodic and in this case approximately elliptic.

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Technical Comments

Comment on "Active Flutter Control Using Generalized Unsteady Aerodynamic Theory"

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RECENTLY Edwards et al.[1] published a method for active flutter control and finite state modelling of aeroelastic systems. While this method is extremely good for two-dimensional airfoils in incompressible and supersonic flow, it is not clear exactly how useful it is in subsonic flow and for three-dimensional lifting surfaces. In the latter case explicit solutions for the pressure and airloads in the complex frequency domain are not available, and the construction of these solutions numerically is not an easy matter. To understand this, one must view the contribution of Ref. 1 in perspective, without all the mathematical trimmings.

In general, the aeroelastic equations in the complex frequency domain are usually written as

$$[MS^2 + CS + K + Q(S)]q = 0 \tag{1}$$

In the method presented in Ref. 2, $Q(S)$ is modelled by rational transfer functions, and this contributes additional states to the finite-state model. On the other hand, these are computed from well-known results for simple harmonic motion of the lifting surface.

In Ref. 1, $Q(S)$ is, in principle, approximated by a polynomial in S; that is

$$Q(S) = P_0S^2 + P_1S + P_2$$

where $P_0 = 0$ for Mach number $\neq 0$, such that the eigenvalues and eigenvectors of the system of coupled equations,

$$[(M+P_0)S^2 + (C+P_1)S + (K+P_2)]q = 0 \tag{2}$$

are identical to those of Eq. (1). This involves the solution of the eigenvalue problem given by Eq. (1), which of course implies that $Q(S)$ must be valid for all S. Thus, in order to construct a finite-state model by the method of Ref. 1, one must not only have calculated the generalized airloads for arbitrary values of the complex frequency S but also solve the complex eigenvalue problem of Eq. (1).

The computation of the unsteady generalized airloads, $Q(S)$, for arbitrary S poses several computational problems and is not an easy task as is assumed in Ref. 1. Furthermore, in order to solve the eigenvalue problem defined by Eq. (1) one must employ fairly sophisticated search techniques.

For three-dimensional lifting surfaces the behavior of the singular induced downwash distributions in the entire complex plane, and not just along the frequency axis, is not clearly known. This problem does not arise for airfoils in incompressible and supersonic flow, which are the only two cases dealt with explicitly in Ref. 1. Thus, in order to extend the unsteady aerodynamics for lifting surfaces in the complex frequency domain, one should first show that singular downwash distributions due to singularities in the unsteady kernel function are uniformly valid in the entire complex plane. Then it is necessary to establish the convergence of the solutions for the pressure to the physically valid solutions just as in the case of numerical methods for solving the lifting surface problem for simple harmonic motions. In the

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literature available thus far no such study has been systematically carried out. The method presented in Ref. 2, it must be pointed out, was meant more for practical three-dimensional lifting surfaces than for airfoils, and to circumvent the above-mentioned problem in calculating generalized airloads in the complex frequency domain.

The method of Ref. 1 for flutter control needs to be refined further so that the relationship between measurements and states can be clearly established in the same fashion as Ref. 2. This is essential if one wishes to construct a reduced-order observer to estimate the states and implement the controller. To see the relationship between measurements and states physically it seems better to work with the system of Eqs. (1), reduce to a set of uncoupled second-order equations, and work back to Eq. (2) rather than reducing them to first order, although the latter is essential for actual computation of the control law. One does not need to work back to Eq. (2) if the relationship between measurements and states is not required as in the case of Ref. 1.

This relationship is especially important for the design of a minimum-order observer (with arbitrary observer dynamics), moreso than eliminating the additional states introduced by the aerodynamic model, as the control law would be a linear functional of only the observer states and measurements. The author's experience has been that such a control law can easily be constructed for a tapered large-aspect-ratio wing with a trailing edge control surface using modified strip analyses, a second-order approximation for Theodorsen's function, and a first-order observer without any measurements of the large number of additional states introduced by the aerodynamic model. A substantial reduction in the required number of measurements was possible when the observer order was increased to two and three. The use of lower order observers seem to be more appropriate than using crude approximations for the aerodynamic model. It is the author's view that the additional states introduced by the aerodynamic models would not only *not* have to be estimated, but would actually make the observer design insensitive to modelling errors – a factor which is certainly important in active flutter control.

Acknowledgments

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Reply by Authors to R. Vepa

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THE application of Laplace transform techniques to aeroelastic vehicle analysis has been studied in Refs. 1-3, where the validity of unsteady aerodynamic theories for arbitrary values of s was demonstrated with examples drawn from two-dimensional incompressible and supersonic flow. In addition, Refs. 1 and 2 presented an active aeroelastic synthesis technique, termed the Rational Model, and gave examples of active flutter control utilizing this technique. Vepa's comments are directed at 1) aeroelastic analysis techniques for arbitrary complex values of s, and 2) the Rational Model synthesis technique.

Aeroelastic systems are modeled as

$$[M_s s^2 + B_s s + K_s - Q(s)]X(s) = G(s)u(s) \tag{1}$$

where M_s, B_s, and K_s are mass, damping, and stiffness matrices, X is an n-dimensional state vector, $G(s)$ is the control distribution matrix, and $Q(s)$ is the unsteady aerodynamic transfer function matrix relating structural motion to generalized forces. The m-dimensional control input vector u is to be interpreted as a position command to aerodynamic control surface servos.

Aeroelastic analysis involves the determination of the roots of Eq. (1) as a function of Mach number and altitude. To this end, the representation of $Q(s)$ may be in any convenient form. Reference 1 demonstrated that the exact complex roots may be determined if $Q(s)$ is known for general values of s and also gave examples of the use of rational function approximations of $Q(s)$ for determining the roots. Such approximations, which Vepa[4] favors for aeroelastic analysis, require augmenting the state of Eq. (1) in order to model the approximations. It is important to understand that $Q(s)$ generally contains nonrational components, such as Bessel functions, which cannot be represented as rational functions (ratios of polynomials in s). Hence, augmented-state models using rational function approximations of $Q(s)$ cannot be exact models, but examples given in Ref. 3 indicate that they may be very adequate for engineering design purposes.

Vepa expresses concern over the validity of unsteady aerodynamic theory for general values of s in subsonic flow and three-dimensional flow. Edwards[5] has addressed this issue and has given examples of calculations of generalized forces in both of these cases. At issue is the applicability of Laplace transform techniques to the governing partial differential equations. It is possible[5] to cast the integral equation solutions into the form of convolution integrals, from which Laplace transformation on the time variable leads directly to the desired generalized aerodynamic solution. A point of confusion in this derivation is the appearance of integrals which are convergent only for $\mathrm{Re}(s) > 0$. This causes no difficulty since the integrals are only representations of the analytic functions describing the physical solution (e.g., Bessel functions) which are valid throughout the s-plane. It is precisely this fact that accounts for the usefulness of rational function approximations of $Q(s)$ and allows their use for unrestricted values of s.

Reference 5 also resolved a point of confusion regarding the calculation of subsonic indicial response functions using the Laplace inversion integral, which may have precipitated some of Vepa's comments.[1] The characteristic starting pulse contained in such functions was shown to be caused by the Kutta wave generated at the trailing edge by the impulsive motion.

Vepa's comments regarding the difficulty of performing analysis and synthesis calculations for arbitrary values of s seem premature since this effort has yet to be attempted in a realistic design case. It seems unfair to compare a new computational technique with long-used techniques which utilize efficient approximations of complex functions.

Whereas aeroelastic analysis may be viewed as the determination of the "open-loop" roots or poles, aeroelastic synthesis is the process of moving these roots to desirable regions of the s-plane. The complexity and robustness of aeroelastic controllers is obviously directly related to the

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